A 20 mV Colpitts Oscillator powered by a thermoelectric generator

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***Abstract*—In this paper, we present a MOSFET-based Colpitts oscillator based on a "zero-threshold" transistor operating at a supply voltage below 20 mV. The circuit was carefully analyzed and expressions relating the start-up conditions and the voltage supply, as well as the oscillation frequency were developed. Measurement results obtained on a discrete prototype confirmed the low-voltage operation of the oscillator, which sustained oscillations of 130 mV (peak-to-peak) at 97 kHz when the voltage supply was 19.8 mV. The circuit was also powered from a thermoelectric generator (TEG) connected to a persons arm in a room with temperature of 24^oC room. Under these conditions, the TEG supplied $22\ mV$ and the circuit operated as expected.**

I. INTRODUCTION

Energy harvesting applications require efficient circuits operating at very low voltages. One emblematic example is the use of a thermoelectric generator (TEG) as the energy source for self-powered wearable medical applications. Bismuth telluride thermoelectric modules provide open-circuit voltages of around 25 mV/K [1]. The small temperature difference between the body and the surroundings, usually of 1 to 2 K, limits the output voltage of TEGs to 50 mV [2], which is a very low voltage level for running conventional electronics.

Some recent publications have reported TEG-based applications operating at supply voltages as low as 20 mV or 35 mV; however, they use DC-DC converters that require special start-up solutions such as pre-charged capacitors or batteries [3] or mechanical switches [2]. The ignition of a DC-DC converter is a time-varying signal commonly obtained from an electronic oscillator. Current state-of-the-art low-voltage oscillators operate at one hundred mV or more [4], [5], imposing a severe limitation to energy scavenging applications.

In this paper, we present results obtained from a Colpitts oscillator powered by a 19.8 mV voltage supply. The circuit uses a "zero threshold" MOS transistor [6] as active element so that the necessary power gain for compensating circuit losses can be obtained from an amplifier powered from a very low voltage supply. In addition to experimental results, we provide an accurate theoretical analysis which allows us to predict the minimum necessary voltage for establishing oscillations as well as the oscillating frequency. The analysis predicts that the oscillator should start-up at a voltage supply of around $13\ mV$, only seven milivolts far from the voltage supply value for which the circuit sustained oscillations at 130 mV(peak-to-peak)/97 kHz. Moreover, the circuit was powered by a thermoelectric generator which supplied a DC voltage of around 22 mV when it was installed on the arm of a person in a 24^oC room.

II. COLPITTS OSCILLATOR

A Colpitts oscillator can be broken in two main blocks : i) a potentially unstable one-port and ii) a load network, as it is shown in Fig. 1. The one-port circuit is an amplifier which can include explicit positive feedback for providing the necessary instability. The load network must be chosen in order to satisfy the Barkhausen criterion, that is to say, the real part of the terminating load must be smaller than the magnitude of the one-port negative conductance, as well as the load susceptance must cancel the one-port imaginary part at the oscillation frequency. Moreover, an amplitude limiting mechanism must be provided so that the amplitude of the oscillations stabilizes at a given point. This mechanism is often obtained from the nonlinear characteristic of the amplifier.

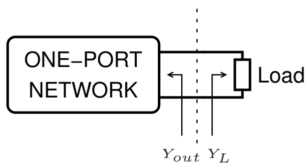

Fig. 1. Conceptual high-level model of a Colpitts Oscillator

In a typical Colpitts implementation, a current source is used to determine the amplifier operating point, however, for extreme low-voltage implementation, the circuit DC path should have only the transistor.

A low-voltage realization of the Colpitts oscillator is presented in Fig 2. The circuit contains a common-gate amplifier, including a tapped capacitor network composed of C_1 and C_2 in order to provide positive feedback. The inductor L_1 is the amplifier load and resonates with the capacitive admittance at the amplifier output. At resonance, the combination of load

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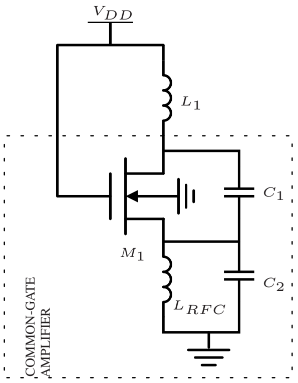

(a)

Fig. 2. Schematic diagram of the ultra-low-voltage Colpitts oscillator.

and amplifier admittance is real and when its sign becomes negative, the oscillations at the transistor drain increase, up to the point at which the nonlinearity of the amplifier alters its bias point, thus stabilizing the signal amplitude. To attain a sufficient level of negative resistance, the amplifier should provide the necessary power gain to compensate for the ohmic losses of all devices. For ultra-low-voltage operation, the terminals of the transistor are connected to the voltage supply (V_{DD}) and to the ground (GND) via inductors L_1 and L_{RFC}.

III. Analysis

In this paper, we are concerned with the minimum voltage necessary to start up the oscillator. Classic analysis of the Colpitts oscillator assumes that the transistor is operating under the saturation condition and disregards the transistor output conductance. Under these conditions, the minimum amplifier voltage gain required for oscillation is found to be 4, when the feedback network capacitors have identical values [7].

For an ultra low-voltage oscillator, a more careful analysis should be conducted, since the transistor operates mainly in the triode region, where its output conductance is considerably higher than in saturation. Considering the AC model of the oscillator shown in Fig. 3.a, we calculated the amplifier output admittance after replacing M_1 with its linear model, as shown in Fig. 3.b. In the model, g_{md} is the drain conductance given by $g_{md} = i_d/v_d$ and g_{ms} is the source transconductance given by $g_{ms} = i_d/v_s$, where i_d is the small-signal drain current and v_d and v_s are the drain and source voltages, respectively [8].

Applying the Kirchoff law of currents in the model of Fig.3(b), we have:

$$i_{out} = v_{out}(j\omega C_1 + g_{md}) - v_s(j\omega C_1 + g_{ms}) \quad (1)$$

$$i_{out} = v_s j\omega C_2 \quad (2)$$

By replacing (2) in (1) and making some algebraic manipulations, we find that the amplifier output admittance is:

$$Y_{out} = \frac{g_{md}\left[1 + \frac{C_1}{C_2}\left(1 - \frac{g_{ms}}{g_{md}}\right)\right]}{\left(1 + \frac{C_1}{C_2}\right)^2 + \left(\frac{g_{ms}}{\omega C_2}\right)^2} + j\frac{\omega C_1\left(1 + \frac{C_1}{C_2}\right) + g_{md}\frac{g_{ms}}{\omega C_2}}{\left(1 + \frac{C_1}{C_2}\right)^2 + \left(\frac{g_{ms}}{\omega C_2}\right)^2} \quad (3)$$

A. Start-up condition

If we consider all the circuit losses due to passive devices embedded in a conductance G_P parallel to L_1, as it is shown in Fig. 4, the oscillator starts up when $Re\{Y_{out}\} < -G_P$:

$$\frac{g_{md}\left[1 + \frac{C_1}{C_2}\left(1 - \frac{g_{ms}}{g_{md}}\right)\right]}{\left[\frac{\omega(C_1+C_2)}{\omega C_2}\right]^2 + \left(\frac{g_{ms}}{\omega C_2}\right)^2} < -G_P \quad (4)$$

Assuming that $g_{ms} << \omega(C_1 + C_2)$, where ω is the oscillator frequency in rad/s, (4) can rewritten as:

$$g_{ms} > g_{md}\left(1 + \frac{C_2}{C_1}\right) + G_P\frac{C_1}{C_2}\left(1 + \frac{C_2}{C_1}\right)^2 \quad (5)$$

B. Minimum voltage supply

In [8] we find an expression relating V_{DS} to the transistor forward and reverse inversion levels, which can be written in terms of the drain and source transconductances as:

$$V_{DD} = \phi_t \ln\frac{g_{ms}}{g_{md}} + \frac{\phi_t^2}{2I_S}(g_{ms} - g_{md}) \quad (6)$$

where ϕ_t is the thermal voltage and I_S is the specific current of the transistor given by $I_S = \mu C'_{ox} n \frac{\phi_t^2}{2}\frac{W}{L}$, μ is the effective mobility, C'_{ox} is the oxide capacitance per unit area, n is the slope factor and W/L is the aspect ratio of the transistor.

In order to establish a relationship with the supply voltage, we should observe in Fig. 1 that V_{DD} is also the drain-to-source DC voltage (V_{DS}) of M_1. So, by combining (5) and (6), we obtain an expression for the minimum supply

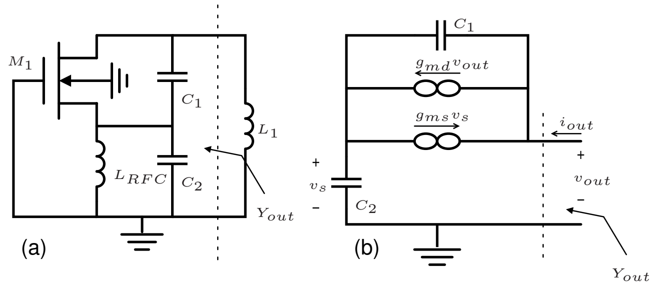

Fig. 3. AC equivalent circuit of the ultra-low-voltage Colpitts oscillator. (a) Schematic (b) Linear model

voltage necessary to start up the MOSFET Colpitts oscillator as follows:

$$V_{DD}|_{crt} = \phi_t \ln\left(1 + \frac{C_2}{C_1}\right) + \phi_t \ln\left[1 + \frac{G_P}{g_{md}}\left(1 + \frac{C_1}{C_2}\right)\right] + g_{md}\frac{\phi_t^2}{2I_S}\frac{C_2}{C_1}\left[1 + \frac{G_P}{g_{md}}\left(1 + \frac{C_1}{C_2}\right)^2\right] \quad (7)$$

The above expression allows us to recognize the importance of the relationship between passive device losses and transistor output conductance. Initially, for a limited supply voltage, increasing the transistor aspect ratio may appear to compensate for the ohmic dissipation in the circuit due to the $\frac{G_P}{g_{md}}$ ratio.

By making $G_P << g_{md}$, a limit for V_{DD} can be given by:

$$V_{DD}|_{G_P<<g_{md}} = \phi_t \ln\left(1 + \frac{C_2}{C_1}\right) + g_{md}\frac{\phi_t^2}{2I_S}\frac{C_2}{C_1} \quad (8)$$

The first term is the V_{DD} absolute limit for deep weak inversion, whereas the second term is added to compensate for the operation in moderate or strong inversion.

By minimizing (7), we find an optimum point for the capacitor ratio C_2/C_1:

$$\left.\frac{C_2}{C_1}\right|_{opt} = \sqrt{\frac{G_P}{(g_{md} + G_P)}} \quad (9)$$

This condition provides a useful starting point for designing the oscillator.

C. Oscillation frequency

The oscillation frequency f_0 is found at the point $Im\{Y_{out}\} = 1/(\omega L_1)$:

$$\frac{\omega C_1\left(1 + \frac{C_1}{C_2}\right) + g_{md}\frac{g_{ms}}{\omega C_2}}{\left(1 + \frac{C_1}{C_2}\right)^2 + \left(\frac{g_{ms}}{\omega C_2}\right)^2} = \frac{1}{\omega L_1} \quad (10)$$

When $g_{ms} << \omega(C_1 + C_2)$ (10) simplifies to

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_T}} \quad (11)$$

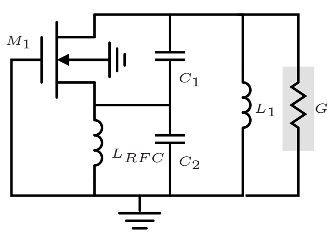

Fig. 4. Oscillator AC equivalent circuit with all passive losses embedded on the conductance G_P

TABLE I
TRANSISTOR CHARACTERISTICS MEASURED FOR $V_{DD} = 20\,mV$ AT $T = 23\,^oC$.

Parameter	Value
I_S (Specific Current)	11.18 μA
V_{TH} (Threshold Voltage)	59 mV
g_{ms} (Source Transconductance)	520 μA/V
g_{md} (Drain Transconductance)	325 μA/V
n (Slope factor)	1.6

where C_T is the equivalent capacitor parallel to L_1, which can be reasonably approximated by $C_T = C_1C_2/(C_1 + C_2)$.

IV. DESIGN

For the design of the oscillator, we used a wide MOSFET with very low threshold voltage and high quality passive devices. The transistor used was a parallel combination of 24 ALD110800 unities [6]. The characteristics of the resulting MOSFET obtained with the aid of a semiconductor parameter analyzer are summarized in Table I.

With the aim of generating a signal at 100 kHz at the lowest supply voltage, we used L_1 and $L_{RFC} \approx$ 10 mH with $Q \approx$ 80@100 kHz and calculated the capacitors for attaining the ratio that minimizes V_{DD}, which resulted in $C_1 \approx$ 2.6 nF and $C_2 \approx$ 520 pF. This value of C_2 considers the necessary compensation for the inductor connected at the transistor source.

For design purposes, we assumed the overall losses due to passive devices to be around twice those of the load inductor. After these considerations, according to (7) we would expect the oscillator to start up with a minimum supply voltage of around 7.7 mV. However, the source and bulk of the transistor used are internally connected. In order to account for the body effect neglected on the previous analysis, we considered the bulk transconductance (g_{mb}) in the model of Fig.3.b. Thus, g_{mb} can be written as $g_{mb} = (g_{ms} - g_{md})(n - 1)/(n)$ [8], where n is 1.6 as shown in Table I. After this correction, the minimum supply voltage is found to be 11.8 mV.

This design was simulated in the ADS Agilent® environment, using the transistor SPICE (LEVEL 2) model obtained from the supplier and passive components including losses measured in an LCR meter. Results from harmonic balance nonlinear simulations predict oscillations for voltage supply values higher than 12 mV, which is very close to the theoretical values. Moreover, for a voltage supply of 20 mV, the oscillations amplitude should be around 160 mV, as it can be seen in Fig. 5.

V. EXPERIMENTAL RESULTS

A prototype was developed for experimental verification using the devices described above. After some tuning, the best results were obtained using $C_1 \approx$ 1.6 nF and $C_2 \approx$ 440 pF, which is easily understood by observing that the transistor

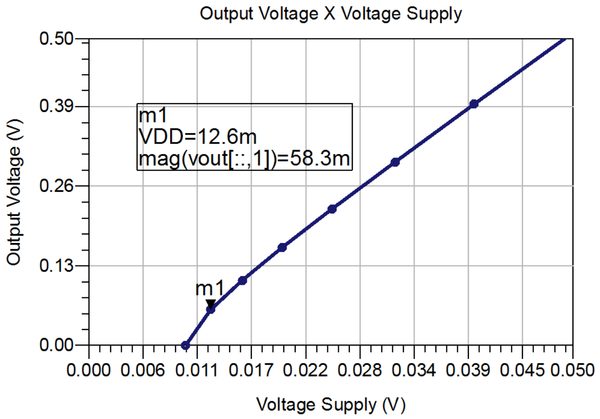

Fig. 5. Output voltage of the oscillator versus voltage supply predicted by harmonic balance simulations

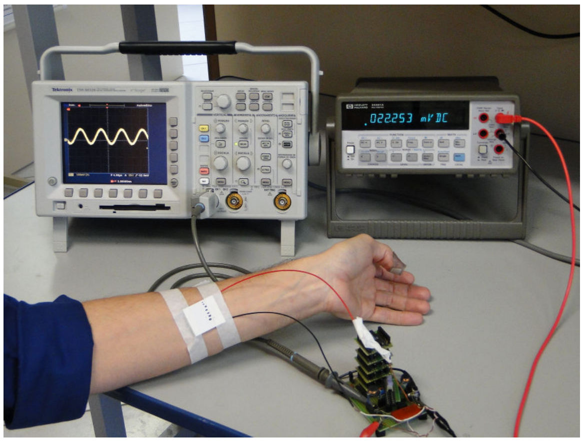

Fig. 7. Photograph of the oscillator working from a thermoelectric generator at T=24 ^{o}C.

parasitic capacitors were not considered in the initial design, as well as the simulation model is not very accurate in the transition from weak to strong inversion operating region.

With these modified values, the circuit sustained oscillations when the voltage supply was around 19.8 mV, only differing by around 7 mV from the value predicted in the analysis. This discrepancy is to be expected since the nonlinearity of the transistor reduces its gain at the point the amplitude is stabilized and the losses were underestimated. The waveform captured with an oscilloscope is shown in Fig.6, where we can observe a 130 mV peak-to-peak signal at around 97 kHz. To the best of the authors' knowledge, this is the the lowest voltage supply ever reported for the operation of a Colpitts oscillator.

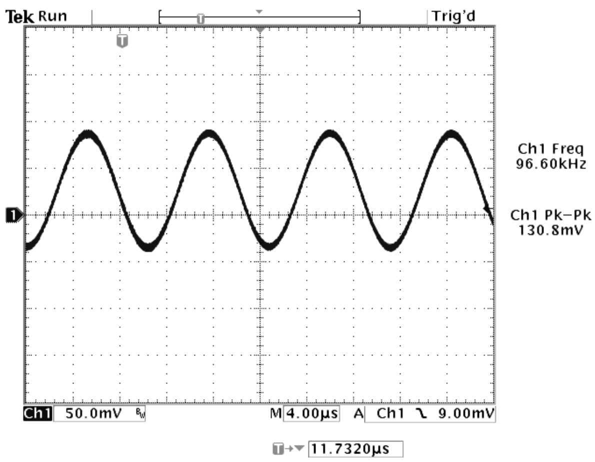

Fig. 6. Waveform of the signal measured at the transistor drain for supply voltage of 19.8 mV at T=23 ^{o}C.

We also did an experiment in which the voltage supply was replaced by a TEG attached to a persons arm. When the room temperature was around 24^oC the TEG output voltage was around 22 mV and the oscillator worked as expected as it can be seen in Fig.7.

VI. Conclusion

In this paper, we presented an ultra-low-voltage Colpitts oscillator, operating at a supply voltage below 20 mV. The circuit was analyzed and start-up conditions were related to voltage supply. Simulations performed using harmonic balance confirmed the analysis. A Colpitts oscillator prototype was developed, based on a very low threshold voltage MOSFET and its operation was confirmed experimentally from a supply voltage below 20 mV, which opens up new opportunities for ultra-low-voltage applications. The circuit was also powered from a thermoelectric generator attached to a persons arm, in an environment with temperature around 24^oC.

VII. Acknowledegements

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